# The Casimir force between dissimilar mirrors and the role of the surface plasmons

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## Abstract

We investigate the Casimir force between two dissimilar plane mirrors the material properties of which are described by Drude or Lorentz models. We calculate analytically the short and long distance asymptote of the force and relate its behavior to the influence of interacting surface plasmons. In particular we discuss conditions under which Casimir repulsion could be achieved.

## I. INTRODUCTION

The availability of modern experimental set-ups that allow accurate measurements of surface forces between macroscopic objects at submicron separations has stimulated increasing interest in the Casimir effect [1]. In his seminal paper Hendrik Casimir calculated the force between two plane parallel perfectly reflecting mirrors in vacuum and found the following expressions for the Casimir force and energy per unit area

$$F_{\mathrm{C}} = -\frac{\hbar c \pi^2}{240 L^4} \quad E_{\mathrm{C}} = -\frac{\hbar c \pi^2}{720 L^3} \tag{1}$$

where sign conventions are such that a negative value of $F_{\mathrm{C}}$ and the energy correspond to attraction between the two plates. $L$ is the mirrors' separation.

The Casimir force between two dielectric plates was first derived in [2]. In the original paper is was mentioned that the spectrum of the problem includes both propagative and evanescent waves. Later, a number of authors [3, 4, 5] succeeded in obtaining the same result through the summation of the modes corresponding to the solutions of the Maxwell equations which decay exponentially in the direction normal to the plates. As a consequence the entire force was interpreted as the interaction of the surface plasmons. However, Schram [6] noticed that the surface modes alone do not yield the right expression. At present we understand that the transition from mode summation to contour integration made in [3, 4, 5, 6], is the most delicate point in the calculation and a possible source of this confusion. The modern point of view is that the surface plasmon interaction dominates at short separations [7, 8, 9]. For plates described by a plasma model or plasma sheets it was shown that the value of the Casimir force is the result of large cancelations between plasmon and photon contributions at all distances[10, 11]. The interaction of the surface modes between a sphere and a plate taking into account the material properties was considered in [12, 13].

In the present paper we study the Casimir force between *dissimilar* plates with dispersion and assess the role of surface plasmons in this case. This set-up gives more possibilities for a modulation of the Casimir force which could be potentially useful in micro- and nanosystems [14]. Besides, dissimilar plates are required for getting a repulsive Casimir force, the realization of which would constitute a major achievement in the field. It is known [15, 16] that the force may change the sign if one of the plates has nontrivial magnetic permeability, $\mu \neq 1$. Novel artificial materials [17, 18] with magnetic response arising from micro- or nanoinclusions have recently been shown to become possible candidates for observing

a repulsive Casimir force[19]. Here we investigate the contribution of the surface plasmon interaction between unequal plates towards a better understanding of the nature of the Casimir repulsion. The general conclusion of our study is that the surface modes should be decoupled as much as possible in order to achieve maximum repulsion.

The outline of the paper is the following. In Section II we calculate the short and long distance asymptotes of the Casimir force for nonequal mirrors. We show that if one of the mirrors has a non-unity magnetic permeability the force is positive at short distances provided the dielectric permittivity is non-unity as well. It is known that at long and medium distances this set-up yields repulsion [20, 21] if one of the mirrors is more magnetic than dielectric. Modeling the dielectric permittivity with a plasma model we find the minimal ratio between magnetic and dielectric plasma frequencies required to get repulsion. It is slightly larger than 1. The case of a purely dielectric mirror facing a purely magnetic one is known to yield the largest possible repulsive force [21, 22] and we consider it separately. There the force has an unusual short distance asymptote, $F \sim 1/L$, that is explained in Section III where we study the plasmon contribution to the force between nonequal mirrors. Section IV contains a discussion of the results and conclusions.

## II. THE CASIMIR FORCE BETWEEN DISSIMILAR MIRRORS

The Casimir force between two flat mirrors separated by a distance $L$ is given by [2, 23]

$$F(L) = -\frac{\hbar}{2\pi^2} \sum_{\rho} \int_0^{\infty} d\kappa \kappa^2 \int_0^{c\kappa} d\omega \, \frac{r_A^\rho r_B^\rho}{\exp(2\kappa L) - r_A^\rho r_B^\rho}. \tag{2}$$

Here $r^\rho(i\omega, \kappa)$, $\rho = TE, TM$, are the reflection coefficients at imaginary frequencies for the mirrors facing vacuum

$$r_i^{TM}(i\omega) = \frac{\kappa_i - \varepsilon_i \kappa}{\kappa_i + \varepsilon_i \kappa}, \quad r_i^{TE}(i\omega) = -\frac{\kappa_i - \mu_i \kappa}{\kappa_i + \mu_i \kappa}, \tag{3}$$

with a dielectric permittivity $\varepsilon_i = \varepsilon_i(i\omega)$, a magnetic permeability $\mu_i = \mu_i(i\omega)$, the imaginary longitudinal wavevector respectively in vacuum and in the material $\kappa = \sqrt{\omega^2/c^2 + k^2}$, $\kappa_i = \sqrt{\omega^2/c^2\,(\varepsilon_i \mu_i - 1) + \kappa^2}$ for mirrors numbered by $i = A, B$.

The properties of the material enter the expression for the Casimir force through the dielectric permittivity and magnetic permeability at imaginary frequencies. From the

Kramers-Kronig causality relation it follows [24, 25], that these functions are always real and positive $\varepsilon(i\omega), \mu(i\omega) \geq 1$. The sign of the force is defined by the sign of the integrand in (2). As $|r(i\omega,\kappa)| \leq 1$, a "mode" $\{\omega,\kappa\}$ gives a repulsive contribution to the force if the corresponding reflection coefficients of the mirrors $A$ and $B$ have opposite signs. This is only possible if the mirrors are different, $r_A \neq r_B$, and if at least one mirror has a nontrivial magnetic permeability. The second condition follows from the analysis of the reflection coefficients (3) at different frequencies.

The Casimir force between plates with a frequency dependent reflectivity is usually calculated numerically by making use of the available optical data or solid state physics models consistent with the Kramers-Kronig relation [23]. Analytical expressions for the Casimir force at short and long distances are known for equal bulk mirrors with dielectric permittivity described by a plasma, Drude or Lorenz models (see for, example [8]) and for mirrors of finite thickness [26]. In what follows we derive analytic expressions for the Casimir force at short and long distances for dissimilar plates described by the models mentioned above.

### A. Short distance asymptote

The material properties define a characteristic length scale $\lambda_{\rm ch}$ for the scattering of the field on the plates and thus also of the system. For example, if the mirrors optical properties are described by a plasma model, this length scale corresponds to the plasma wavelength. We therefore define the short distance range with respect to this characteristic wave-length of the permittivity model, $L \ll \lambda_{\rm ch}$. As the main contribution to the integral over $\kappa$ (2) comes from $\kappa \sim 1/L$, we deduce that only wave-vectors $1/\kappa << \lambda_{\rm ch}$ contribute essentially to the Casimir force. At short distances we may approximate the wave-vector

$$\kappa \sim k + \frac{1}{2}\frac{\rho_i^2}{k} + \dots \tag{4}$$

where $\rho_i^2 \equiv (\varepsilon_i\mu_i - 1)\omega^2/c^2$, $i = A, B$. The force is then given by

$$F = -\sum_\rho \frac{\hbar}{2\pi^2}\int_0^\infty dk\, k^2 \int_0^\infty d\omega \frac{r_A^\rho r_B^\rho\, e^{-2kL}}{1 - r_A^\rho r_B^\rho\, e^{-2kL}} \tag{5}$$

$$r_i^{TM}(i\omega) = \frac{1-\varepsilon_i}{1+\varepsilon_i} + \frac{\varepsilon_i}{(1+\varepsilon_i)^2}\frac{\rho_i^2}{k^2} + \dots,$$

$$r_i^{TE}(i\omega) = \frac{1-\mu_i}{1+\mu_i} + \frac{\mu_i}{(1+\mu_i)^2}\frac{\rho_i^2}{k^2} + \dots. \tag{6}$$

Thus the magnetic/dielectric properties of the material show up in the second therm of the expansion of the transverse magnetic/electric reflection coefficient (6). This term is usually omitted. However, here we will take it into account in order to determine the sign of the force when $\varepsilon$ or $\mu$ are equal to unity and the leading term vanishes.

Let the material be described by the general model

$$\begin{aligned}\varepsilon(\omega) &= 1 - \frac{\omega_e^2}{\omega^2 - \omega_0^2 + i\Omega_e\omega} \equiv 1 + \frac{\omega_e^2}{\omega^2}\, B_e(\omega)\\ \mu(\omega) &= 1 - \frac{\omega_m^2}{\omega^2 - \omega_0^2 + i\Omega_m\omega} \equiv 1 + \frac{\omega_m^2}{\omega^2}\, B_m(\omega),\end{aligned} \tag{7}$$

which reduces to the well known plasma model for $\Omega_e = \omega_0 = 0$ with a plasma frequency $\omega_e$ and to the Drude model with $\omega_0 = 0$ where $\Omega_e$ describes the electronic relaxation frequency. Then the leading terms of the short distance asymptote for the reflection coefficients are given by

$$\begin{aligned}r^{TM}(i\omega) &= -\frac{\omega_e^2}{2(\omega^2 + \omega_0^2 + \Omega_e\omega) + \omega_e^2},\\ r^{TE}(i\omega) &= \frac{\omega_m^2}{2(\omega^2 + \omega_0^2 + \Omega_m\omega) + \omega_m^2}.\end{aligned} \tag{8}$$

The absorbtion in the material has the strongest influence at low frequencies. On the contrary, in the short distance range mainly the high frequencies make the decisive contribution to the force. Therefore we may neglect the absorbtion in the material for short distances, by putting $\Omega_e = \Omega_m = 0$.

First we calculate the short distance limit for the force between a purely magnetic mirror A facing a purely dielectric mirror B. When $\varepsilon = 1$, the leading term in $r^{TM}$ vanishes, $\rho^2 = \omega_m^2 B_m(i\omega)/c^2$, and

$$r^{TM}(i\omega) = \frac{\omega_m^2}{4\,c^2}\,\frac{\omega^2}{k^2}\,\frac{1}{\omega^2 + \omega_0^2 + \Omega_m\omega}. \tag{9}$$

To assess the TM contribution, we therefore use the reflection coefficients $r_A^{TM}$ defined by (8) and $r_B^{TM}$ defined by (9).

In the same manner, when $\mu = 1$, then the leading term in $r^{TE}$ vanishes, $\rho^2 = \omega_e^2 B_e(i\omega)/c^2$ and

$$r^{TE}(i\omega) = -\frac{\omega_e^2}{4\,c^2}\,\frac{\omega^2}{k^2}\,\frac{1}{\omega^2 + \omega_0^2 + \Omega_e\omega}. \tag{10}$$

For the contribution coming from the TE modes, the reflection coefficients $r_B^{TE}$ given by expression (10), and $r_A^{TE}$ defined by eqn. (8) must be employed.

We then evaluate the force for the Drude model, i.e. $\omega_0 = 0$. The TM contribution reads

$$F^{TM} = \frac{\hbar\sqrt{2}\,\omega_{mA}^2\,\omega_{eB}}{32\pi c^2\,L}\int_0^\infty dk \frac{k\,e^{-2k}}{\sqrt{k^2 + \frac{\omega_{mA}^2 L^2}{8c^2}\,e^{-2k}}}. \tag{11}$$

where we can neglect the second term in the denominator, as $\omega_{mA}^2 L^2/4c^2 << 1$. The TM contribution to the Casimir force then simplifies to

$$F^{TM} \approx \frac{\sqrt{2}}{64}\,\frac{\hbar}{\pi c^2}\,\frac{\omega_{mA}^2\omega_{eB}}{L}. \tag{12}$$

The contribution of TE modes is calculated in an analogous way, replacing $\omega_{mA}$ in (12) by $\omega_{eB}$. This leads to the following expression for the total Casimir force

$$F \approx \frac{\sqrt{2}}{64}\,\frac{\hbar}{\pi c^2}\,\frac{(\omega_{eB}^2\,\omega_{mA} + \omega_{mA}^2\,\omega_{eB})}{L}. \tag{13}$$

The force is repulsive and has an unusual short distance asymptote $\sim 1/L$. This unusual behavior will be explained in the next section.

Now we evaluate how the introduction of a small dielectric permittivity for the former purely magnetic mirror B affects the Casimir force. To this aim we calculate the short distance limit for the force between a magneto-dielectric mirror A facing a purely dielectric mirror B. In this case the TE contribution at short distances is negligible with respect to the TM one. The TM contribution to the Casimir force has now to be evaluated using the TM reflection coefficients (8). After expanding the integrand in (5) and integration over $k$ we obtain $F = -H_{AB}/3L^3$, where $H_{AB}$ is sometimes refereed in the literature as non-retarded Hamaker constant [27]. At zero temperature it is

$$H_{AB} \;=\; \frac{3\hbar}{8\pi^2}\int\limits_0^\infty d\omega \sum_{n=1}^\infty \frac{(r_A^{TM}[i\omega]\,r_B^{TM}[i\omega])^n}{n^3}. \tag{14}$$

Further we recast (14) in order to see the dependence of force on the parameters more clearly. If we rewrite the reflection coefficients (8) as

$$r_i^{TM}(i\omega) = -\frac{\Omega_{1,i}^2}{\omega^2 + \Omega_{2,i}^2} = -\frac{\Omega_{1,i}^2}{\Omega_{2,i}^2}\frac{\Omega_{2,i}^2}{\omega^2 + \Omega_{2,i}^2}, \quad i = A, B,$$

where $\Omega_{1,i}^2 = \omega_{e,i}^2/2$, and $\Omega_{2,i}^2 = \omega_{e,i}^2/2 + \omega_0^2$, the integrand simplifies. The result of integration over $\omega$ may be expressed in terms of the hypergeometric series.

Finally we obtain the Casimir force at short distances

$$F \;\simeq\; -\frac{\hbar}{L^3}\frac{\Omega_{2B}}{16\pi^2}\sum_{k=0}^\infty \frac{\Gamma(k+\frac{1}{2})}{k!}\,G_k\,\left(1 - \frac{\Omega_{2B}^2}{\Omega_{2A}^2}\right)^k, \tag{15}$$

where

$$G_k = \sum_{n=1}^{\infty} \frac{\Gamma(n+k)\,\Gamma(2n-\frac{1}{2})}{\Gamma(n)\,\Gamma(2n+k)\,n^3} \left(\frac{\Omega_{1A}}{\Omega_{2A}}\frac{\Omega_{1B}}{\Omega_{2B}}\right)^{2n}. \tag{16}$$

If the plates are described by a plasma model we have $\omega_{0A} = \omega_{0B} = 0$, $\Omega_{1A} = \Omega_{2A}$, $\Omega_{1B} = \Omega_{2B}$ while the numerical coefficients (16) do not depend on the plasma frequencies of the mirrors. The Casimir force then writes

$$F \simeq -\frac{\hbar}{16\pi^2 L^3}\frac{\omega_{e,B}}{\sqrt{2}}\,\gamma,$$

where, for example, $\gamma \approx 1.744$ for two equal mirrors. If, in contrast, the plasma frequency of mirror A is slightly higher, $\omega_{e,B}/\omega_{e,A} = 0.9$, the force increases, and $\gamma \approx 1.836$. The result does not depend on the magnetic properties of the mirror A.

Our analysis has shown that the Casimir force between a mirror A with $\mu_B \neq 1$ and a purely dielectric mirror B is always attractive in the short distance range and determined by the modes corresponding to the TM polarization of the electromagnetic field. When $\varepsilon_A \to 1$ the TM contribution becomes small and comparable to the contribution of the TE modes. Only in this case the force is repulsive at short distances and has the form defined in Eq.(13)

### B. Long distance asymptote

In order to find the condition for the occurrence of a repulsive Casimir force we add here the analysis of the long distance limit between the two mirrors ($L >> \lambda_{eA}, \lambda_{eB}$). For two dissimilar purely dielectric mirrors described by a plasma model the long distance limit is obtained by expanding the integrand of (2) in powers of the small parameter $\lambda_{eA}/L$ or $\lambda_{eB}/L$

$$F|_{L>>\lambda_{eA},\lambda_{eB}} = \eta\,F_C(L), \quad \eta \approx 1 - \frac{4}{3\pi}\frac{\lambda_{eA}+\lambda_{eB}}{L}. \tag{17}$$

where we have introduced the force reduction factor $\eta$[23]. For a magneto-dielectric mirror A described by a plasma model with plasma frequencies $\omega_{mA}, \omega_{eA}$ in front of a purely dielectric mirror B, characterized by a plasma frequency $\omega_{eB}$, the force reduction factor tends at long distances to

$$\eta(\alpha) \to \frac{180}{\pi^4}\frac{3}{8}\left\{\alpha\int_0^{1/\alpha} d\Omega\,\mathrm{Li}_4\left(\frac{\Omega-1}{\Omega+1}\right) + \frac{1}{\alpha}\int_0^{\alpha} d\Omega \sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}\left(\frac{\Omega-1}{\Omega+1}\right)^n\right\} \tag{18}$$

$\alpha$ gives the ration between the magnetic and dielectric plasma frequency of mirror A $\alpha = \omega_{mA}/\omega_{eA}$. In contrast to the case of two dielectric mirrors, the long distance limit of the reduction factor between a magneto-dielectric and a pure dielectric mirror is not 1. In particular the sign of the Casimir force is defined by the parameter $\alpha$. As the leading term (18) of the long distance asymptote does not depend on the plasma frequency of the nonmagnetic mirror B, we can estimate the ratio between the dielectric and magnetic plasma frequencies of the plate A, required to get a repulsive force. A numerical analysis shows that for a value of $\alpha_0 \approx 1.0255$ the long distance asymptote of the Casimir force vanishes. In other words, within the plasma model the Casimir repulsion is achieved when $\alpha > \alpha_0 = 1.0255$. The ratio is lower than the one obtained in [15] for *constant* dielectric permittivity and magnetic permeability, $\mu \sim 1.08\varepsilon$, provided mirror B is a perfect conductor.

On the other hand, when mirror A has equal dielectric and magnetic responses, that is $\alpha = 1$, the force is positive at any plate separation and the long distance limit of the reduction factor is given by

$$\eta(1) = \frac{180}{\pi^4}\frac{3}{64}\int_0^1 d\Omega \, \mathrm{Li}_4\left(\left(\frac{\Omega-1}{\Omega+1}\right)^2\right) = 0.0205$$

Finally, for a purely dielectric mirror A facing a purely magnetic mirror B we obtain the long distance limit characterized by strong repulsion

$$F|_{L>>\lambda_{eA},\lambda_{eB}} = \eta\, F_{Cas}(L), \quad \eta \approx -\frac{7}{8} + \frac{7}{6\pi}\frac{\lambda_{eA}+\lambda_{mB}}{L}. \tag{19}$$

The repulsive force given by the first term of this expansion was first obtained by Boyer [22] for two non-dispersive mirrors with $\epsilon_A = \infty$, $\mu_A = 1$ and $\varepsilon_B = 1$, $\mu_B = \infty$.

## III. DESCRIPTION IN TERMS OF INTERACTING SURFACE PLASMONS

In this section we will analyze the attractive and repulsive behavior of the Casimir force taking into account the differences in the surface plasmon coupling for the different combinations of materials. Eq. (2) for the Casimir force includes both photon and plasmon contributions that have different signs. In [10] it was shown that the value of the Casimir energy is the result of the compensation between the photon and plasmon contributions for equal mirrors described by plasma model. Moreover at short distances the force is entirely defined by the attraction of the surface plasmons. The analysis of the photon and

plasmon contributions for dissimilar mirrors, especially if they have a nontrivial magnetic permeability, is more complicated. That is why we restrict ourselves to the plasma model for the material properties of the mirrors. We start from the general definition of surface plasmon energy. As in the previous section, we first consider the plasmon interaction for two dissimilar purely dielectric mirrors, then for magneto-dielectric mirrors and finally we discuss a purely dielectric mirror facing a purely magnetic one.

Between two arbitrary plates the surface plasmon modes exist for both field polarizations, TE and TM. Their frequencies $\omega_\sigma^\rho$ are implicitly defined as the solutions of the equations

$$\prod_{i=A,B} \frac{\kappa_i + \varepsilon_i q}{\kappa_i - \varepsilon_i q} = e^{-2\,q\,L}, \qquad \prod_{i=A,B} \frac{\kappa_i + \mu_i q}{\kappa_i - \mu_i q} = e^{-2\,q\,L} \tag{20}$$

with $q^2 = k^2 - \omega^2/c^2 \geq 0,\ \kappa_i^2 = k^2 - \varepsilon_i \mu_i\, \omega^2/c^2, \quad i = A, B$. The left and right equations refer respectively to the TM and TE polarization. Both equations have two different solutions which are numbered by the index $\sigma$.

The vacuum energy of the interacting surface plasmons living on the plane mirrors is then formally given by

$$E^{sp} = \frac{\hbar}{2} \sum_{\rho,\,\sigma} \int\limits_{k_{(\sigma)}}^{\infty} \frac{dk\,k}{2\pi}\, [\omega_\sigma^\rho]_{L\to\infty}^{L}\,, \quad \rho = TE, TM. \tag{21}$$

The infinite energy of single-surface plasmons has been subtracted in this expression as it corresponds to an infinite plate separation. When both mirrors are equal, the two TM surface plasmons which one obtains as solutions of (20) are called symmetric and antisymmetric plasmons [10, 11] or binding and anti-binding resonances [9]. Each solution exists for $k > k_{(\sigma)}$.

### A. Two dissimilar purely dielectric mirrors

Let us first consider two dissimilar dielectric mirrors described by a plasma model

$$\varepsilon_i(\omega) = 1 - \frac{\omega_{e,i}^2}{\omega^2}, \quad \mu_i(\omega) = 1,\ i = A, B$$

with $\omega_{eB}/\omega_{eA} = \beta_e$. In TM polarization on each mirror lives a single-surface plasmon with a frequency

$$\omega_i^{sp} = \frac{1}{\sqrt{2}} \left[ \omega_{ei}^2 + 2|k|^2 c^2 - \sqrt{\omega_{ei}^4 + 4k^4 c^4} \right]^{1/2} \tag{22}$$

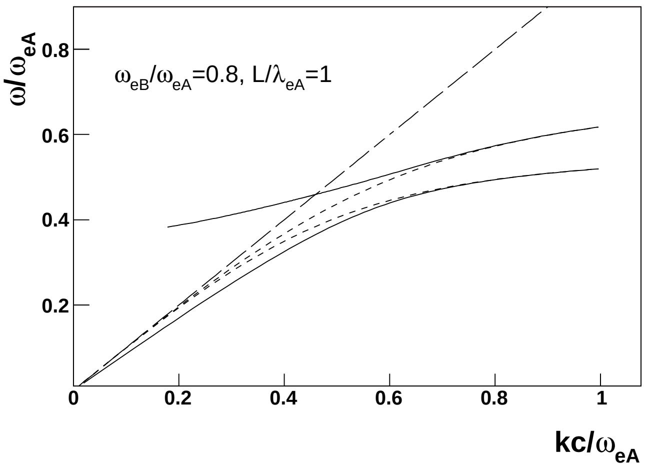


FIG. 1: Solid curves: the TM plasmon modes which are the solution of (20); dashed curves: noninteracting single-surface plasmons, (22); long-dashed line: boundary between propagative and evanescent sectors

When both mirrors become close to each other they start to interact according to TM equation (20). It has two solutions. In analogy with the case of two equal mirrors we call them symmetric and antisymmetric plasmons. Let $\omega_{eB} \leq \omega_{eA}$ ($\beta_e \leq 1$). At large plate separations the frequency of the symmetric plasmon, $\omega_+$, tends to the frequency of the unperturbed surface plasmon of mirror A $\omega_A^{sp}$, while the frequency of the antisymmetric one, $\omega_-$, approaches the frequency of the unperturbed plasmon of mirror B, $\omega_B^{sp}$. The behavior of the plasmon modes being the solutions of (20) is plotted on Fig. 1 for $L/\lambda_{eA} = 1$ and $\beta_e = 0.8$. The straight line $\omega = kc$ marks the limit between the sector of propagating waves $kc < \omega$ and evanescent waves $kc > \omega$. The plasmon $\omega_+$ crosses the boundary between the propagative and evanescent sectors when

$$k \equiv k_{(+)} = \frac{\omega_{eA}}{c}\sqrt{\frac{\beta_e(\beta_e+1)}{1+\beta_e(\Lambda+1)}}, \quad \Lambda = \omega_{eA}L/c. \tag{23}$$

The plasmon mode $\omega_-$ lies entirely in the evanescent sector, $k_{(-)} = 0$.

In [10] and the subsequent papers [28, 29] an adiabatic mode definition was used which attributes the entire mode $\omega_+$ to the evanescent sector. Mathematically it is equivalent to putting $k_{(+)} = 0$. This choice gives a repulsive plasmon contribution to the Casimir force while the photon contribution is attractive, and allows to recover the perfect mirrors limit with an attractive force coming alone from the photonic mode contribution corresponding to

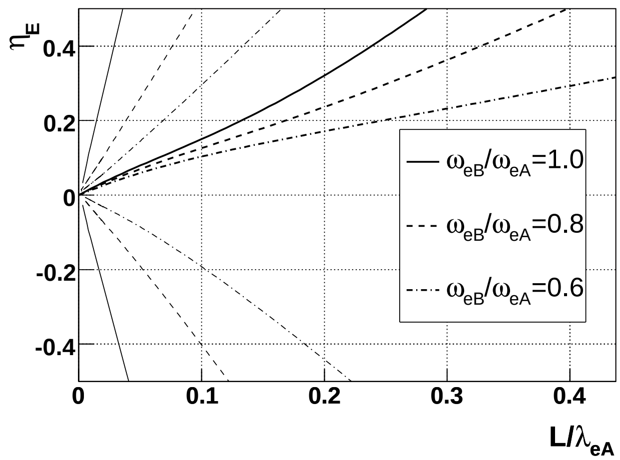


FIG. 2: Thick solid, dashed and dash-dotted lines: the normalized contribution of the surface plasmons to Casimir energy for different ratios $\omega_{eB}/\omega_{eA}$. The attractive contribution of $\omega_-$ and repulsive contribution of $\omega_+$ to $\eta_{pl} = E^{sp}/E_C$ are traced in thin lines.

the original derivation by H. Casimir[1]. For simplicity we will adopt here the convention of [11, 30] and attribute the two parts of the symmetric surface plasmon to the corresponding sectors. When calculating its vacuum energy we start the integration over $k$ from the value given in (23).

Fig. 2 shows a plot of the surface plasmon contribution to the energy for different values of $\beta$. The total plasmon energy is negative corresponding to an attractive force (positive $\eta_{pl}$) and comprises the repulsive contribution of the symmetric plasmon and the attractive contribution of the antisymmetric plasmon. The attraction between the surface plasmons is the strongest when the plasma frequencies of the plates coincide.

Fig. 3 shows plasmon and photon contributions to the Casimir energy as a function of the normalized distance $L/\lambda_{eA}$ between the plates for three different ratios of the electric plasma frequencies, namely $\omega_{eB}/\omega_{eA} = 1$ (long dashed and dotted-dashed), $\omega_{eB}/\omega_{eA} = 0.8$ (medium dashed and dotted dashed) and $\omega_{eB}/\omega_{eA} = 0.6$ (short dashed and dotted dashed). When the plates are unequal, $\omega_{eA} \neq \omega_{eB}$, the balance between the plasmon and photon contribution to the energy obviously changes. Each contribution on its own decreases and so does the total Casimir energy. As the plasma frequencies do not match, the plasmon interaction is weaker and the respective contribution to the energy is smaller.

On the other hand, the photon contribution is reduced as well. The transparency of the

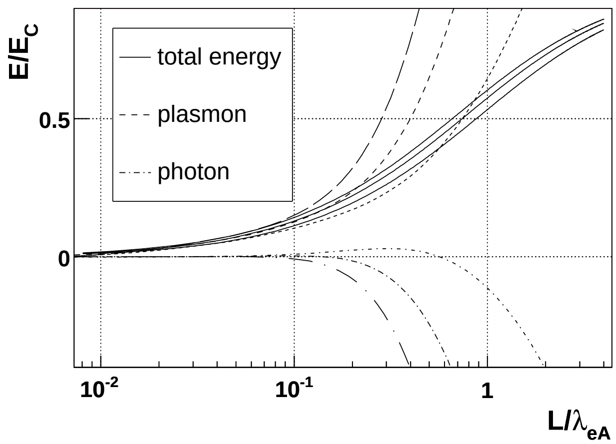


FIG. 3: The Casimir energy for the dissimilar mirrors as a result of cancelation between photon and plasmon contributions. The plasmon contributions for $\omega_{eB}/\omega_{eA} = 1, 0.8, 0.6$ are plotted respectively by long, medium, and short dash lines. The dash-dotted curves give the corresponding photon contribution, and the solid lines give the total Casimir energy. The total energy is the largest for equal mirrors, $\alpha = 1$, the lowest plot is the total energy for $\alpha = 0.6$.

mirror for the propagating waves is governed by its plasma frequency. If $\omega_{eA} > \omega_{eB}$, the fluctuations from the band $\omega_{eB} < \omega < \omega_{eA}$ are reflected by the plate A, but pass through the plate B. Thus the high frequency cut-off for the cavity formed by both mirrors is defined by the plasma frequency of the less reflecting plate B. Consequently the distance depending part of the vacuum energy which comes from the propagative sector has a smaller value than in the case of plates with equal plasma frequencies $\omega_{eA}$, resulting in a smaller Casimir force.

The long distance asymptote of the plasmon contribution, $\Omega_{e,i} \equiv L\omega_{e,i}/c \gg 1$ may be calculated explicitly. To this end, we first introduce the dimensionless variables $K = kL, \Omega = \omega L/c$, and $\Omega_{e,i} = \omega_{e,i}L/c$ in (21,20) leading to

$$E^{sp} = \frac{\hbar c}{4\pi L^3}\left\{\int\limits_{K_+}^{\infty} dKK(\Omega_+ - \Omega_A^{sp}) + \int\limits_{0}^{\infty} dKK(\Omega_- - \Omega_B^{sp})\right\}. \tag{24}$$

Then we change the integration variable in (24), $K \to Q = \sqrt{K^2 - \Omega^2}$ and write the renormalized energies of the symmetric and antisymmetric plasmons as

$$E^{sp}|_{\Omega_{eB}\gg 1} = \frac{\hbar c}{4\pi L^3}\left\{\int\limits_{0}^{\infty} dQQ\{\Omega_- + \Omega_+ - \Omega_B^{sp} - \Omega_A^{sp}\},\right.$$

$$+\int_0^{Q_A} dQ\, Q\, \Omega_A^{sp} + \frac{1}{3}(\Omega_+^3 - \Omega_A^3)|_{K_{(+)}}^{\infty} \Bigg\} . \tag{25}$$

If the plasma frequencies of the plates are close to each other, the following simplifications can be performed

$$\begin{aligned}
&\Omega_\pm = \sqrt{\frac{\Omega_{eB}\, Q}{2\beta}}\sqrt{\frac{1+e^{-2Q}}{1-e^{-2Q}}}\, f_\pm^{1/2}(\beta_e, Q), \quad 0<\beta\leq 1,\\
&f_\pm(\beta, Q) = (1+\beta_e) \pm \sqrt{(1-\beta)^2 + \frac{16\beta e^{-2Q}}{(1+e^{-2Q})^2}},\\
&\Omega_A^{sp} = \sqrt{\beta^{-1}\Omega_{eB} Q}, \quad \Omega_B^{sp} = \sqrt{\Omega_{eB} Q},\\
&K_{(+)} = \sqrt{1+\beta^{-1}}\sqrt{\Omega_{pB}},\\
&Q_A = \sqrt{K_{(+)}^2 - \Omega_A^{sp}(K_{(+)})} \to 1+\beta,
\end{aligned}$$

With these we find a long distance approximation for the vacuum energy of the interacting surface plasmons

$$E_{sp}|_{\Omega_{eB}\gg 1} = \frac{\hbar c\sqrt{\Omega_{eB}}}{4\sqrt{\beta_e}\pi L^3}\psi(\beta) = \frac{\hbar\sqrt{c\omega_{eA}}}{4\pi L^{5/2}}\psi(\beta_e). \tag{26}$$

For the different values of $\beta_e$ plotted in Fig.3 we find $\psi(0.6) \approx -0.0663$, $\psi(0.8) \approx -0.163$ and $\psi(1) = -0.2798$. The last value, corresponding to equal mirrors, confirms the result published in [11]. It is important to note that the accuracy of the long distance approximation (26) becomes worse as $\beta$ decreases.

The situation when one of the plates is perfectly conducting corresponds to $\beta_e \ll 1$ (but $\Omega_{eB}, \Omega_{eA} >> 1$). In this case the anti-binding surface plasmon $\omega_+$ dominates at large distances. The total energy of the surface plasmons becomes positive, yielding repulsion.

### B. Two dissimilar magneto-dielectric mirrors

To assess under which conditions Casimir repulsion may arise we will finally investigate the situation of two dissimilar magneto-dielectric plates. Let the dielectric permittivity and magnetic permeability of the mirrors be described by a plasma model

$$\varepsilon_i(\omega) = 1 - \frac{\omega_{e,i}^2}{\omega^2}, \quad \mu_i(\omega) = 1 - \frac{\omega_{m,i}^2}{\omega^2}, \; i = A, B$$

Then the frequencies of the single surface plasmons of the respective mirror are given by

$$\omega_{sp,i}^{TM} = \frac{\omega_{e,i}}{\sqrt{2}} \left[ 1 - \frac{2k^2c^2}{\omega_m^2 - \omega_e^2} - \sqrt{1 + \frac{4k^4c^4}{(\omega_m^2 - \omega_e^2)^2}} \right]_i^{1/2} . \tag{27}$$

When $k \to 0$ the single surface plasmon frequency vanishes, while it tends to $\omega_{e,i}/\sqrt{2}$ when $k \to \infty$. If $\omega_m = \omega_e \equiv \omega_p$, the frequency of the single surface plasmon does not depend on $k$ and $\omega_{sp,i}^{TM} \to \omega_{p,i}/\sqrt{2}$. In the limit of small plate separations, corresponding to large wave vectors, the equations for the interacting surface plasmons may be solved explicitly:

$$(\omega_{\pm}^{TM})^2 = \frac{\omega_{eA}^2 + \omega_{eB}^2}{4} \mp \sqrt{\frac{(\omega_{eA}^2 - \omega_{eB}^2)^2}{16} + \frac{\omega_{eA}^2\omega_{eB}^2}{4} e^{-2|k|L}} \tag{28}$$

To obtain the solutions corresponding to the TE surface plasmons one has to replace $\omega_{e,i}$ by $\omega_{m,i}$ for both mirrors $i = A, B$.

Substituting (28) into (21) we may derive the vacuum energy of the surface plasmons at short plate separation. It is given by

$$E_{pl}(L) = \frac{\hbar}{16\pi L^2} \left[ \frac{\omega_{eA}}{\sqrt{2}} \chi(\beta_e) + \frac{\omega_{mA}}{\sqrt{2}} \chi(\beta_m) \right] , \tag{29}$$

where the parameters $\beta_e = \omega_{eB}/\omega_{eA}$ and $\beta_m = \omega_{mB}/\omega_{mA}$ are introduced. $\chi(z)$ is given by the following expression

$$\chi(z) = \int_0^\infty dkk \left\{ \left[ \left( z_+^2 + \sqrt{z_-^4 + z^2\, e^{-k}} \right)^{1/2} - 1 \right] + \left[ \left( z_+^2 - \sqrt{z_-^4 + z^2\, e^{-k}} \right)^{1/2} - z \right] \right\} \tag{30}$$

with $z_+^2 = (1 + z^2)/2$ and $z_-^2 = (1 - z^2)/2$. For a positive argument the function $\chi$ is negative and varies from 0 to $\chi(\infty) \to -0.1358$. Thus the energy of interacting surface plasmons is always negative yielding attraction between unequal magneto-dielectric mirrors. Reformulating the argument in terms of the Casimir force, $F_{pl} = -dE_{pl}/dL$, it means that magneto-dielectric plates allow only for an attractive Casimir force at short distances, which is the result of the interaction of the surface plasmons.

The limit $z \to \infty$ corresponds to vanishing frequencies $\omega_{eA}$ or $\omega_{mA}$ and hence to vanishing TE and TM plasmon contributions to the Casimir energy. For equal mirrors we find $z = 1$ and $\chi(1) \simeq -0.2776$, corresponding to the maximum value of the Casimir energy.

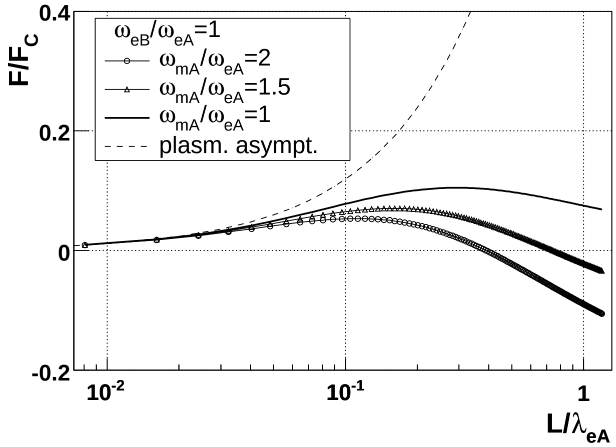


FIG. 4: Reduction factor $\eta_F = F/F_C$ at short plate separations as a function of dimensionless distance $\Lambda = 2\pi L/\lambda_{eA}$. The dashed line is the short distance asymptote of the surface plasmon interaction.

In Fig.4 we finally show the result for the force correction factor $\eta_F(L/\lambda_{eA}) = F/F_C$, evaluated numerically as a function of the distance normalized by $\lambda_{eA} = 2\pi c/\omega_{eA}$. The force is calculated between two plates with $\varepsilon_A(\omega) = \varepsilon_B(\omega) = 1 - \omega_{eA}^2/\omega^2$. Plate B is purely dielectric. The magnetic permeability of plate A is $\mu_A(\omega) = 1 - \omega_{mA}^2/\omega^2$. We vary the magnetic response of the plate A and calculate the force making use of the exact formula (5). The total correction factor $\eta_F$ is strongly affected by $\omega_{mA}$ and becomes negative at medium and long distances if $\omega_{mA} > \omega_{eA}$, which indicates a repulsive Casimir force. The TM plasmons exist on both plates, while the TE plasmons live only on plate A. Thus their energy does not depend on the plate separation and they do not influence the force.

The plot also shows the short distance asymptote of the plasmon contribution (dashed line), which is linear in $\Lambda = L/\lambda_{eA}$, $\eta_F^{pl} \simeq 1.1933\,\Lambda$ in agreement with [23]. It does not depend on the magnetic permeability of plate A.

Finally we discuss the particular case of a purely dielectric mirror facing purely magnetic one. From (27) one can easily see that the mirror has no TM plasmon modes if it is purely magnetic ($\omega_{eA}^2 = 0$). Similarly if the mirror is purely dielectric ($\omega_{mB}^2 = 0$) it has no TE plasmon modes. Imagine a hypothetic Casimir set-up with a purely dielectric mirror placed in front of a purely magnetic one. Then the existing TM-plasmon mode of purely dielectric mirror and TE-pasmon mode of the purely magnetic mirror are not coupled.

Indeed, in this particular case the solution (28) does not depend on the distance between the plates , $\omega_+^{TM} = 0$, $\omega_-^{TM} = \omega_{eB}/\sqrt{2}$ and $\omega_+^{TE} = \omega_{mA}/\sqrt{2}$, $\omega_-^{TE} = 0$. The vacuum energy is distance independent as well. Therefore the plasmon modes do not contribute to the Casimir force. The force is repulsive and entirely defined by the propagating modes. This explains the unusual short distance asymptote (15).

We have studied the interaction of the surface modes for the magneto-dielectric plates described by the simplest plasma model. Surface polaritons living on the *single* interface of two media, one of which is left-handed with the magnetic permeability described by alternative model, was examined in [31]. The interaction of these modes within the Casimir cavity is of interest.

## IV. DISCUSSION AND CONCLUSIONS

In the present paper we have considered two dissimilar plates with frequency dependent dielectric permittivity and magnetic permeability. Analytic expressions for the force at short and long distances were derived. They give us the the sign of the force and its dependence on the parameters of the solid state physics models describing the material of the plates. To explain these results we have studied the vacuum energy of the interacting surface modes living on the mirrors. We confirm the conclusion formulated in [10, 11] for equal mirrors described by plasma model, that at all distances the value of the force is the result of large cancelation between the plasmon and photon contributions. The problem for dissimilar mirrors appeared to be more complicated. Varying the properties of plates affects both propagative and evanescent modes. However our analysis indicates that provided one of the mirrors is mainly magnetic the balance between the plasmon and photon contributions is shifted to repulsion and we have determined the ratio between the dielectric and magnetic plasma frequencies required to obtain repulsion. The general conclusion of our study is that in order to achieve a repulsive Casimir force, the surface plasmons should be decoupled as much possible. The limiting, though experimentally not achievable, case corresponds to a purely dielectric mirror facing a purely magnetic one. For this situation we explain the repulsion at all distances by the complete absence of the interaction between the surface modes.

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